Arnoldy based passive model order reduction algorithm.

Igor Balk *

Abstract: Many problems in modern interconnect and packaging design require use of 'full-wave' modeling in order to get a good accuracy at high frequencies. In this paper we will introduce passive Arnoldy based model order reduction algorithm for such systems.

1. Introduction

The topic of model order reduction appears in many disciplines. In control systems, perhaps the best known approaches are Truncated Balanced realizations, Hankel singular values. Other popular approaches are based on the modal properties of the system, such as method of Selective Modal Analysis, which is widely used in the power systems applications. Unfortunately, most of this method require $O(n^3)$ computation since they require explicit knowledge of the entire eigenspectrum of the system as in selective modal analysis or the hankel singular values as in truncated balanced realization. For complicated packaging and interconnect problems for which n can be quite large, such methods would require days and gigabytes of memory to compute.

In area of circuit simulation, asymptotic waveform evaluation has popularized the use of model order reduction. A robust approach of deriving moment matching reduced order model is Arnoldi process. The Arnoldy process has it's origins in eigenvalues computation but was used recently to generate moment matching reduced order models.

In this paper we will give brief overview of Arnoldy based moment matching algorithm of model order reduction and will introduce new guarantee passive Arnoldy based algorithm for reduction 'full-wave' models.

2. Overview of Arnoldy based methods of model order reduction

Lets consider single input – single output linear system:

$$A\frac{dx}{dt} = x + bu \qquad (1.1)$$
$$y = c^t x$$

A here is m x m matrix and $b, c \in R^m$.

From (3.1) we can write that relation for transfer function H(s) in Laplace space

$$H(s) = -c^t (I - sA)^{-1} b \qquad (1.2)$$

Now we can expand our transfer function H(s) in a Taylor series around zero frequency.

$$H(s) = \sum_{i=0}^{\infty} m_i s^i = \sum_{i=0}^{\infty} -c^t A^{-(i+1)} b s^i \qquad (1.3)$$

• Igor Balk is with Intellisense Corp. 36 Jonspin Road, Wilmington, MA 01887 fax: 978 988 8001 e-mail: ibalk@intellisense.com

where m_i , the coefficient of the i-th term in the Taylor series, is known as the i-th moment of the transfer function. Most commonly used model order reduction algorithm utilizes Arnoldy method of fast generation of orthogonal basis for Krylov subspace defined as

$$K_q(A,b) = span\{b, Ab, A^2 b ... A^{q-1} b\} \tag{1.4}$$

The general idea of this method is to generate k-th order ortogonalized Krylov subspace from (k-1)-th order orthogonalized Krylov subspace. The last vector of (k-1)-th order orthogonalized subspace is multiplied by A and then the resulting vector is orthogonalized with respect to the rest of the basis. After n steps this algorithm returns a set of n orthogonal vectors and we can construct matrix $V_n \in R^{m \times n}$ and n x n upper Hessenberg matrix H_n such as

$$AV_n = V_n H_n + h_{n+1,n} v_{n+1} e_n^t \tag{1.5}$$

Then we can easy see that for k<n

$$A^k b = \|b\|_2 A^k V_n e_1 = \|b\|_2 V_n H_n^k e_1 \tag{1.6}$$

With this relation we can write that moments can be related to H_n by

$$m_k = c^t A^k b = \|b\|_2 c^t V_n H_n^k e_1 = c_n^t A_n^k b_n \tag{1.7}$$

and so on and n-th order Arnoldi based approximation to H(s) can be written as

$$H(s) \approx \|b\| c^t V_n (I - sH_n)^{-1} e_1 \tag{1.8}$$

corresponding to the steady state representation $A_n = H_n$, $b_n = e_1$ and $c_n = \|b\| V_n c^t$.
And substituting it for the representation of original system we will obtain that

$$\begin{aligned} V_n^t A V_n \frac{dx}{dt} &= V_n^t x + V_n^t bu \\ \tilde{y} &= c^t V_n^t x \end{aligned} \tag{1.9}$$

And

$$y \approx \tilde{y} \tag{1.10}$$

3. Passive algorithm for reduction of ‘full-wave’ models

Now lets assume that A is frequency dependent matrix and that transfer function is positive definite for all s with non negative real part i.e. $\mathrm{Re}(i, H(s)i) \geq e$ for some $e > 0$. This kind of problem quite common in modeling of interconnect circuits or packaging devices at high frequencies when we have to use ‘full-wave’ kernel to generate integral equation formulation. Standard Arnoldy algorithm is unable to handle such kind of problems and there was a lot of work done in this field. We will introduce here passive algorithm for Arnoldy based reduction of such systems.

In order to generate reduced order model we will present A(s) in a from:

$$A(s) = \frac{1}{s^p + 1} A_t + \frac{s^p}{s^p + 1} A_t \tag{2.1}$$

where A_i is power series expansion of A(s) with infinite radii of convergence. And we will generate first order infinite dimension matrix in way similar to the described in [1].

$$\left[I + \frac{s}{s^p+1} \begin{bmatrix} \hat{A}_0 & \hat{A}_1 & \cdots & \hat{A}_N \\ -I & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & -I & 0 \end{bmatrix} \right] \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} b \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$y = [c^t \quad 0 \quad \cdots \quad 0] \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_N \end{bmatrix} \tag{2.2}$$

where $x_k = s^k x$ and $\hat{A}_k = A_0^{-1} A_{k+1}$ for k+1<p and $\hat{A}_k = A_0^{-1}(A_{k+1} + A_{k+1-p})$ for k+1≥ p .

And now we can use Arnoldy process for the new systems with $s = \frac{s}{s^p+1}$. We can formalize described algorithm as follows

1. **Calculate** A_0
2. **set** n**=0 and set some** p**>1**
3. **Calculate zero order transfer function**
4. n++
5. **Calculate** A_n
6. **Update** p **such as** $\left| s \max_{i,j} \hat{A}_{i,j} \right| < 1$ **for any** A_n **and** s
7. **Update** A_l **for all** l **from** 0 **to** n
8. **Calculate** n**-th order transfer function using (1.3)**
9. **Check convergence of transfer function**
10. **If converge calculate reduced model else back to step 4**

Using the properties of power series expansion with infinite radii of convergence and definition of passivity it's possible to show that introduced algorithm will produce passive reduced model. Proof is based on the fact that we do not have to truncate expansion of A(s) in order to calculate n-th order of reduced model and that expansion of transfer function will converge for any value of s **.**

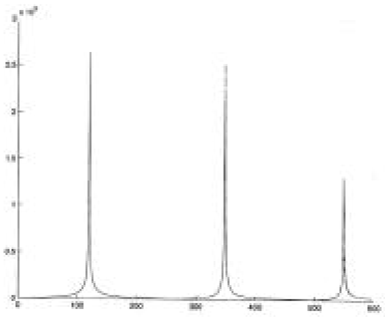
4.Some computational results

Now we will take a look in to some computational results. To demonstrate the method, consider of 2-D transmission line.

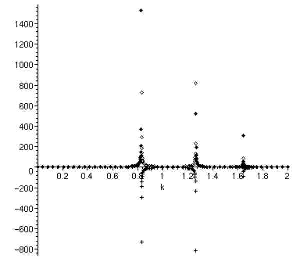
On a picture 1.1 we can see transmission line made from the copper conductors which are 5 cm long and 1 microns wide and separated by 10 microns. It's descretized in a way which produces N=7200 original unknowns and reduced to the system with the number of unknowns n=70. Expansion of 50 order is used to represent 'full wave' kernel. Solid line represents impedance of the original system and dashed line – of the reduced one.

As we can see from the pictures we can fierily well use described algorithm to reduce order of the model in two orders of magnitude which gives us huge saving in computational time and memory required.

Now we will compare described algorithm with known Taylor series expansion method. On a figure 1.2 we can see behavior of transfer function of transmission line function for passive (diamonds) and non passive (crosses) cases. Passive model was generated by described algorithm and non passive one by using of Taylor series of same order.



Picture 1.1. Solution of system of 7200 unknowns (solid line) and solution of reduced system of 70 unknowns (dashed line). On x-axes frequency f*10^5 Hz, on y-axes admittance



Picture 1.2. Guarantee passive (diamonds) and Taylor expansion based (crosses) reduced models. On x-axes normalized frequency, on y-axes normalized admittance

5. Bibliography

[1] Joel R. Phillips, Eli Chiprout and David D. Ling. "Efficient full-wave electromagnetic analysis via model order reduction of fast integral transforms". 33rd DAC, Las Vegas 1996

[2] L. Miguel Silveira, Mattan Kamon and Jacob White. "Efficient reduced-order modeling of frequency-dependent coupling inductances associated with 3-D interconnect structures" .
32rd DAC San Francisco, 1995

[3] Kevin J. Kerns, Ivan L. Wemple and Andrew T Wang. "Stable and efficient reduction of substrate model networks using congruent transform". ICCAD San Jose, 1995

[4] Luis Miguel Silveira. "Model order reduction techniques for circuit simulation". Ph.D. thesis. MIT, Cambridge, 1994

[5] E.J.Grimme, D.C. Soensen and P. van Dooren. "Model reduction of state space systems via an implicitly restarted Lanczos method". Number. Algorithms, 1995

[6] A. Odabasioglu, M. Celik, and L. Pileggi. "PRIMA: passive reduced order interconnect macromodeling algorithm" IEEE conference on computer-aided design, San Jose, 1997