Stability Criterion For Microscale Concentric Flow Of Two Immiscible Liquids

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ABSTRACT

This paper addresses the establishment of a stability criterion for isothermal laminar concentric flow of two immiscible liquids. Using the approach of combined non-dimensional group analysis and numerical simulation, it is established that the inner core liquid will be stable and maintain a thread like configuration if the capillary number exceeds unity, but it will be unstable and break up into droplets if the capillary number is below unity.

The methodology presented can also be extended to flow in different geometry and regimes.

Keywords: immiscible, microfluidics, stability, simulation, multifluid

INTRODUCTION

Concentric flow of two immiscible fluids has many important engineering applications. In pipeline systems where crude oil is being transported, a non-mixing shielding fluid is often introduced as a lubricant [1]. In microscale ink jet applications, a stream of liquid is broken up into droplets in an ambient fluid (air) [2-4]. One potential application of microscale concentric flow of two immiscible liquids is DNA sequencing, where the focusing of a core liquid is performed with the use of an encapsulating sheath liquid.

For multifluid flows, stability consideration is of paramount importance, since it addresses the question of whether or not a desired flow configuration is physically realizable. In macroscale flows such as those in pipeline systems, a desired stable flow is achieved by using a shielding liquid, which has a lower viscosity than the core liquid [1,5]. Conversely, in microscale flows such as those in ink jet applications, flow instability is exploited as a mechanism for droplet generation.

In the present study we considered an axisymmetric flow cell in which an immiscible sheath liquid is introduced and encapsulates an inner core liquid, see Figure 1. The radius of the core liquid can then be adjusted by varying the volume flow rates of the two liquids. For this operation, instability is detrimental since it prevents the formation of a steady stream of core liquid in the form of a thread.

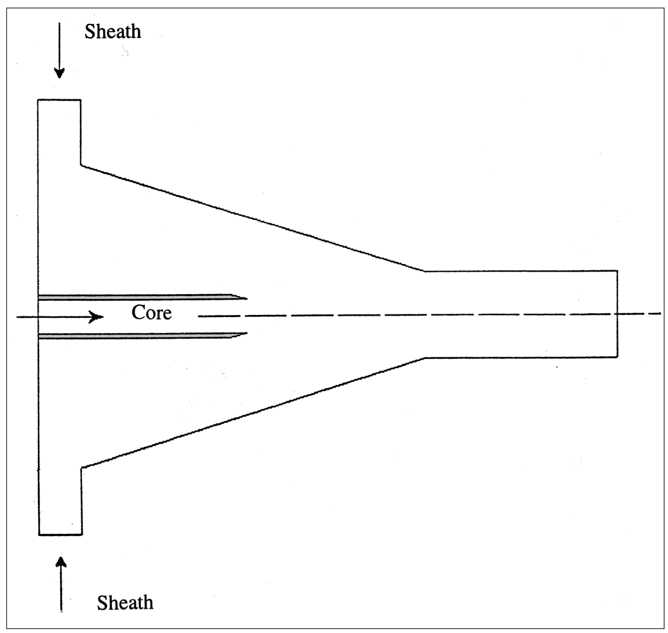

Figure 1: Device geometry

For design and analysis purposes, accurate methods are required to predict whether or not the flow is stable under a given set of flow parameters. While experimental studies are extremely valuable, the fabrication of a large number of device prototypes can be costly; therefore, this approach is not cost effective in the initial design stage. Instead, fluidic

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analysis would be more effective in developing an optimized design before prototyping.

In the present approach, first a study of non-dimensional groups is performed in order to identify the relative importance of the various physical effects that govern the flow stability (Analysis section). Next, a summary is given of an extension of numerical algorithm developed for free surface flows for the simulation of two immiscible with a well-defined interface (Numerical Simulation section). Then, the simulation results are presented from which a criterion for flow stability is established (Results and Discussions section). Lastly, the conclusion of this study is given (Conclusion section).

ANALYSIS

In the present analysis, it is assumed that the flow is both isothermal and incompressible. The geometry of the axisymmetric flow model is illustrated in Figure 1. The inner tubing through which the core liquid is injected has an inner diameter of 30 μm and a wall thickness of 5 μm, while the diameter of the outlet varies between 200-400 μm. Typical combined volume flow rate at the outlet is between $4X10^{-5}$ to $4X10^{-4}$ cm^3/s. The flow is assumed to exit the chamber at constant ambient pressure.

Regarding fluid properties, a nominal value of the interface tension coefficient is selected based on a combination of various candidates of core and sheath liquids. Other liquid properties are assumed to be similar to those of water. Table 1 gives the characteristic flow quantities to be used for the analysis below.

Table 1: Characteristic Quantities

Quantity	**Notation**	**Value**
Density	ρ	1 gm/cm^3
Viscosity	μ	0.01 gm/cm-s
Interface Tension Coef.	σ	20 dyn/cm
Length	L	0.02 cm
Velocity	V	0.3 cm/s

Note that these quantities are derived from an examination of the isothermal incompressible Navier-Stokes equation together with the interfacial and external boundary conditions that govern the flow.

Next, four non-dimensional groups are evaluated which relate the relative dominance of various physical effects that influence the flows. These effects include viscosity, inertia, interfacial tension, and gravity. The groups are defined in Table 2, together with estimates computed using the numerical values of characteristic quantities given in Table 1.

Table 2: Nondimensional Groups Considered

Parameter	**Notation**	**Definition**	**Value**
Reynolds Number	Re	$\rho VL/\mu$	0.6
Capillary Number	Ca	$\mu V/\sigma$	0.0002
Ohnesorge Number	Oh	$(\rho\sigma L)^{1/2}/\mu$	63
Bond Number	Bo	$\sigma/(\rho g L^2)$	51

Note that other nondimensional groups can be constructed from these four. For example, the Weber Number (W) which relates inertia to interfacial tension effects is a product of the Reynolds Number, which relates inertia to viscous effects, and the Capillary Number, which relates viscous to interfacial tension effects (that is, W = Re Ca). Also, in the Bond Number definition *g* is the acceleration due to gravity.

As indicated by the numerical value, a low Reynolds Number flow implies that viscous effects are more significant than inertia effects. An examination of the other three groups that involve interface tension effects shows that interfacial surface tension dominates over all other effects. Previous studies show that viscosity effects tend to stabilize the flow while surface tension effects tend to destabilize it [6]. These results suggest for the flow under consideration that a criterion for flow stability should be established based on both interface tension and viscous effects.

Both the Capillary Number and the Ohnesorge Number are possible candidates for the establishment of a stable flow criterion. In a previous study, the breakup of a cylindrical thread of liquid into droplets in an infinite ambient liquid is considered in Ref. [7], in which the Ohnesorge Number is suggested as the governing parameter. However, for microscale coating flows [8-9], the Capillary Number is identified as one of the key governing parameters.

In order to establish a criterion applicable to the present study, other tools are required both to establish the relevant parameter and a critical value where transition from a stable flow to an unstable flow occurs. In the next section, we present the use of computational fluid mechanics to provide simulation results from which a stability criterion can be established.

NUMERCIAL SIMULATION

The numerical algorithm employed is an extension of the free-surface formulation and algorithm given in Ref. [10]; a summary follows. The formulation uses an Arbitary-Lagrangian-Eulerian description, in which a mesh velocity is introduced in the weak form of the Navier-Stokes equation.

In the computation of the mesh velocity, the time evolution of the core liquid-sheath liquid interface is tracked accurately using the Lagrangian description, while an elliptic operator is used to extend the mesh velocity from the interface to the interior to minimize mesh distortion. In the imposition of the interfacial tension contribution, integration-by-parts is performed on the product of the interface tension coefficient and the in-plane curvature, which is a corollary of the treatment given in Ref. [11]. As a result, the slope instead of the curvature appears in the weak form of the interface tension contribution. The spatial discretization of the governing equations employs high-order finite elements where a typical polynomial order of 4-6 is used in the simulation. A semi-implicit time integration is employed in which the mesh evolution as well as the convection contributions are treated explicitly.

In the flow simulation, a transient solution is performed for each given set of flow parameters, using the steady state solution with a flat interface as initial conditions. A typical stable flow configuration is shown in Figure 2, in which the core liquid maintains a thread-like configuration. A typical unstable flow configuration is shown in Figure 3, from which the progressive formation of a droplet is evident.

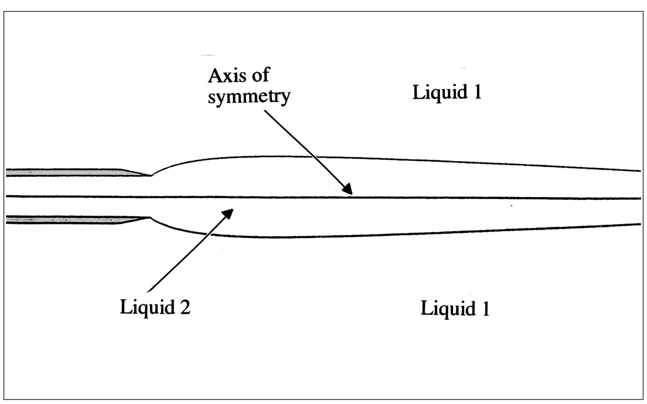

Figure 2: Stable flow profile

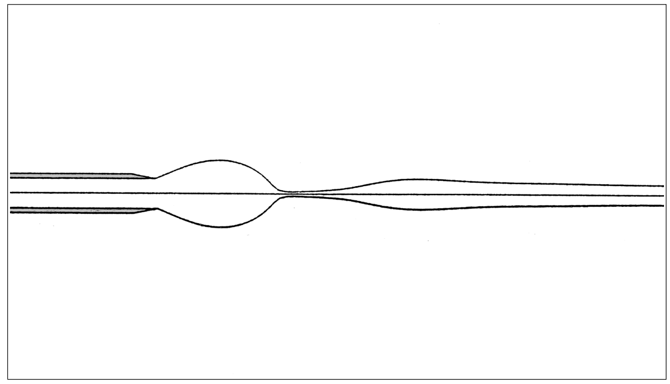

Figure 3: Unstable flow profile

RESULTS AND DISCUSSIONS

Numerical simulations were performed for flow parameters that correspond to various values of Ohnesorge and Capillary Numbers. The results are tabulated in Table 3. The Reynolds number in these simulations ranges from 0.04 to 4.

Table 3: Prediction of Stability of Flow Based on Numerical Simulation

Case	Capillary No.	Ohnesorge No.	Result
A	0.12	1.8	Unstable
B	0.12	0.566	Unstable
C	1.2	1.8	Stable
D	1.2	0.566	Stable

A comparison of cases A & C and B & D shows that for the same value of Ohnesorge Number, the flow can be either stable or unstable. This suggests that the Ohnesorge Number is not a relevant parameter for stability determination in the present case.

The numerical results also suggest that for the range of flow parameters considered, stability of the flow can be quantified by whether the Capillary Number is above or below unity. $Ca > 1$ will give a stable flow while $Ca < 1$ will give an unstable flow.

Additional simulations were performed to study cases in which the core and sheath liquids have different viscosity. Ratios of the core viscosity to sheath viscosity from 1/2 to 2 were considered. It was found that for this range of viscosity ratio, the difference in viscosity did not alter the above stability criterion.

The established stability criterion can be used for flows which are similar; that is, with comparable values of the non-dimensional groups.

In addition, the methodology presented can also be applied for design and analysis of flow systems with different flow parameters.

CONCLUSION

The onset of the break up of a core stream of liquid into droplets was investigated by performing a combined study of nondimensional groups and a parametric study through numerical flow simulation. For the range of flow parameters under consideration, the Capillary number has been established as the key parameter that governs flow stability, with the critical value of unity. Thus, flows with $Ca < 1$ will tend to break up into droplets while flows with $Ca > 1$ will tend to maintain a thread-like configuration.

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REFERENCES

[1] Joseph, D.D., and Renardy, Y.Y., Fundamentals of two-fluid dynamics Part II, Springer Verlag, IAM Vol. 4, 1993.

[2] Radev, S. and Tchavdarov, B., "Linear capillary instability of compound jets," Int. J. Multiphase Flow, Vol. 14, pp.67-79, 1988.

[3] Verdonckt-Vandebroek, S., "Micromachining technology for thermal ink jet products," SPIE Proceedings, Vol. 3224, 1997.

[4] Tseng, F.G., Kim, C.J. and Ho, C.M., "A novel microinjector with virtual chamber neck," 11th MEMS Proceedings, 1998.

[5] Joseph, D.D., Nguyen, K. and Renardy, Y., "Instability of the flow of two immiscible liquids with different viscosities in a pipe," J. Fluid Mech, Vol. 141, pp. 309-317, 1984.

[6] Drazin, P.G. and Reid, W.H., Hydrodynamic Stability, Cambridge Univ Press, 1981.

[7] Lee, W.K. and Flumerfelt, R.W., "Instability of stationary and uniformly moving cylindrical fluid bodies," Int. J. of Multiphase Flow, Vol. 7, pp.363-383, 1981.

[8] Christodoulou, K.N. and Scriven, L.E., "The fluid mechanics of slide coating," J. Fluid Mech., Vol. 208, pp.321-354, 1989.

[9] Coyle, D.J., Macosko, C.W. and Scriven, L.E., "Stability of symmetric film-splitting between counter-rotating cylinders," J. Fluid Mech., Vol.218, pp.437-458, 1990.

[10] Ho, L.W. and Patera, A.T., "A Legendre spectral element method for simulation of unsteady incompressible viscous free-surface flow," Comp. Meth. Appl, Mech. Eng., Vol.80, pp.355-366, 1990.

[11] Ho, L.W. and Patera, A.T., "Variational formulation of three-dimensional viscous free-surface flows: natural imposition of surface tension boundary conditions," Int. J. Num. Meth. Fluids, Vol.13, pp.691-698, 1991.