Effects of squeezed film damping on dynamic finite element analyses of MEMS

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ABSTRACT

The use of micro-fluidics in MEMS device design and implementation is becoming a common practice, especially in the area of biomedical devices. As with all MEMS devices, a validated analysis tool can be invaluable in the design and development process. One of the more common manifestations of micro-fluidics in MEMS design is in the area of squeezed film damping. When one surface moves in close proximity to another solid surface, the fluid in the space between the moving surface and the solid surface can have a significant effect on the dynamics of the moving plate. If the fluid flow in the gap can be assumed to be quasi-steady and viscous-dominant, and if the gap height is small compared to the plate width, the velocity profile of the fluid between the moving plate and the solid surface can be approximated as parabolic in the thickness direction. In this case, the Navier-Stokes equations governing the fluid flow can be reduced to a scalar equation in terms of the fluid pressure. This squeezed film model can be combined with a solid mechanical (finite-element) model in order to perform dynamic fluid-structure interaction analyses for cases in which the above assumptions are valid. In such an analysis, the 3-dimensional solid mechanical model will provide the plate geometry, location, and velocity to the squeezed film model, which will in turn provide the resulting fluid pressure on the moving plate to the solid mechanical model. In this paper, cases will be presented in which the stated assumptions will be validated, and the relevant equations will be derived for both compressible and incompressible fluids. Examples of solid and perforated plates moving in compressible and incompressible fluids will be provided, and their results will be verified against fluid dynamics theory.

Keywords: Squeezed-film damping, micro-fluidics, dynamic analysis, CFD, CAD, FEA, MEMS

1. INTRODUCTION

MEMS design and analysis software products were developed to address the specific needs of MEMS engineers. Fabrication process-based model creation, anisotropic etch simulation, thin film material property characterization, fully coupled 3-dimensional thermo-electromechanical analysis, and micro-assembly analysis are examples of capabilities that have been developed to allow modeling of more complex MEMS designs [1,2]. As MEMS technology progresses, new product families become feasible, and new software capabilities are required to simulate the fabrication and operation of these new devices.

The importance of micro-fluidics in MEMS device design, analysis, and operation has been growing for several years. The interaction between MEMS devices and their environmental fluid—liquid or gas—has always been a major design consideration in the field of BioMEMS. Recently, however, the effects of various micro-fluidic phenomena, including squeezed film damping, have become extremely important in the design of other MEMS devices, including but not limited to sensors, RF switches, and MOEMS.

Squeezed film damping is a term used to describe one of the more common fluid-structure interactions that impacts the performance of MEMS devices. Squeezed film damping occurs when a plate moves in close proximity to another solid surface, in effect alternately stretching and squeezing any fluid that may be present in the space between the moving plate and the solid surface. This fluid can act as a spring and/or a dashpot, having a significant effect on the dynamics of the moving plate. Figure 1 shows an example of such a device, with a porous moving plate.

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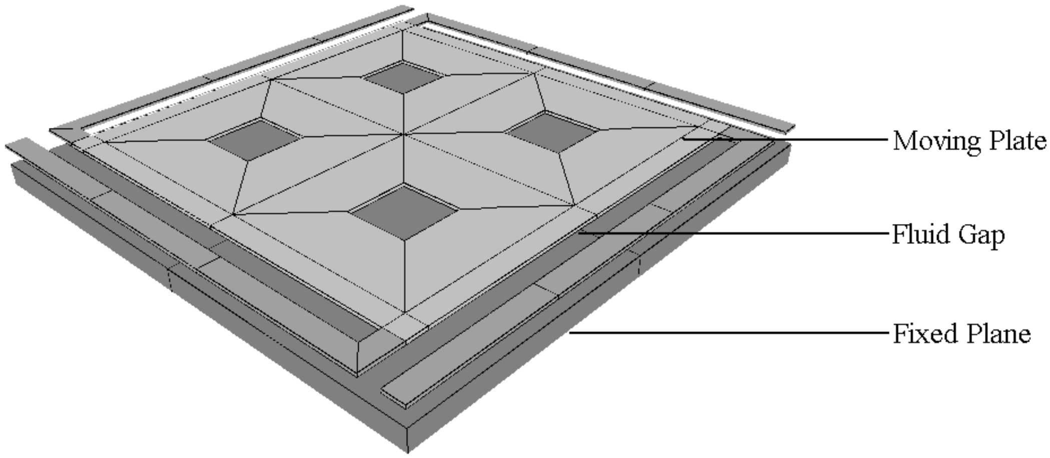

Figure 1: Porous plate moving relative to a fixed solid surface

The true benefit of using a MEMS design and analysis software package is obtaining information about the fabrication and performance of a device without a potentially costly and time-consuming microfabrication iteration. This benefit increases as the accuracy of the results generated by these computer simulations improves. By accurately and efficiently capturing the effect of a fluidic environment on a device at the analysis stage, a MEMS developer can decrease the overall time and cost of the product development cycle.

2. ASSUMPTIONS

To maximize the accuracy of the calculated squeezed film damping effects while minimizing the computational expense required to achieve a solution, a few assumptions are made. The flow must be assumed to be quasi-steady. Second, the flow must be assumed to be viscous-dominant. Finally, the gap height, *h*, must be small compared to the overall width of the plate. The accuracy of any analysis performed involving squeezed film damping will depend on the validity of these assumptions for the flow inside the gap.

For the quasi-steady flow assumption to be applicable, the characteristic time of the fluid in the gap, t_f, should be small compared with the time period of the motion of the solid structure. The characteristic time of the fluid in the gap is defined as

$$t_{\mathrm{f}} = \rho h^2 / \mu \, , \qquad \text{Eq. 2.1}$$

where ρ is the density of the fluid.

The second assumption is that the viscous term of the full Navier-Stokes equation dominates the convection term. That is, the Reynolds number, *Re*, is small.

$$Re = \frac{\rho V h}{\mu} \, , \qquad \text{Eq. 2.2}$$

where ρ is the density of the fluid, V is the velocity of the moving plate, and μ is the viscosity of the fluid.

The final assumption required is that the gap height is small compared to the overall width of the plate. In general, it is best to ensure that

$$h \leq 0.1(L), \qquad \text{Eq. 2.3}$$

where L is the overall width of the moving plate.

3. DERIVATION OF EQUATIONS

The effects of squeezed film damping on the dynamic response of a MEMS device are manifested as a pressure distribution across the face of the moving plate. To determine the magnitude of this pressure as a function of time and position, begin with the two dimensional Navier-Stokes equations:

$$-\frac{dp}{dx} + \mu \frac{d^2 u}{dz^2} = 0 \qquad \text{Eq. 3.1}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial z}(\rho w) = 0, \qquad \text{Eq. 3.2}$$

where p is the pressure, t is time, x is lateral position, z is vertical position, u is the lateral velocity of the fluid, and w is the vertical velocity of the fluid. Figure 2 is presented to display the variables of interest.

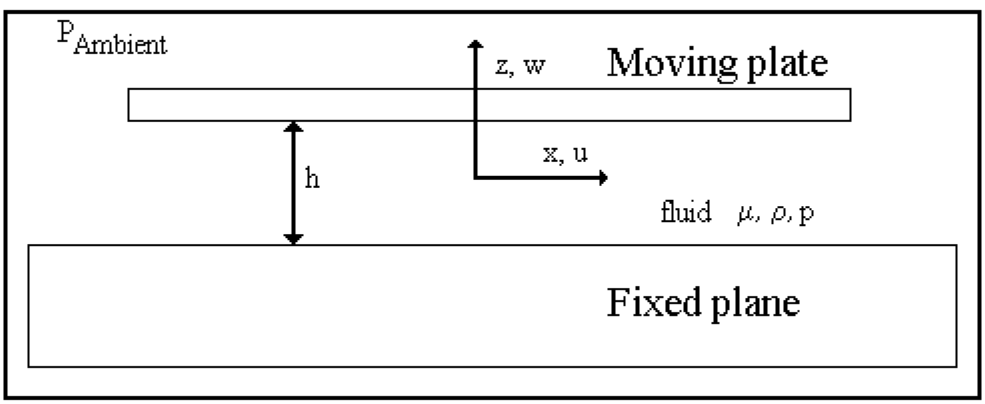

Figure 2: Squeezed film model variables

By integrating Equation 3.1 with respect to z (across the fluid gap), the following relationship is obtained:

$$u = \frac{1}{2\mu} \frac{dp}{dx} \left(z^2 - zh\right). \qquad \text{Eq. 3.3}$$

Substituting Equation 3.3 into Equation 3.2 and again integrating with respect to z across the fluid gap,

$$\int_0^h \frac{\partial}{\partial x}\left[\rho \frac{1}{2\mu} \frac{d\rho}{dx}\left(z^2 - zh\right)\right] dz = -\int_0^h \frac{\partial \rho}{\partial t} dz - \int_0^h \frac{\partial}{\partial z}(\rho w) dz. \qquad \text{Eq. 3.4}$$

However, it has been stated that h is the height of the fluid gap, and w is the vertical velocity of the fluid. Therefore,

$$w\big|_{z=0} = 0 \text{ and } w\big|_{z=h} \equiv \dot{h} = \frac{dh}{dt}, \qquad \text{Eq. 3.5}$$

and thus

$$\frac{d}{dx}\left[\frac{\rho}{2\mu}\frac{d\rho}{dx}\left(-\frac{h^3}{6}\right)\right] = -h\frac{\partial \rho}{\partial t} - \rho\frac{dh}{dt}. \qquad \text{Eq. 3.6}$$

Equation 3.6 can be simplified to

$$\frac{d}{dx}\left[\frac{h^3}{12\mu}\rho\left(\frac{d\rho}{dx}\right)\right] = \frac{d}{dt}(\rho h). \qquad \text{Eq. 3.7}$$

Equation 3.7 represents the two-dimensional solution. The fully three-dimensional solution is obtained as:

$$\nabla \bullet \left[\frac{h^3 \rho}{12\mu}(\nabla \rho)\right] = \frac{d}{dt}(\rho h). \qquad \text{Eq. 3.8}$$

3.1 COMPRESSIBLE FLUID

For a compressible fluid under isothermal conditions, pressure can be substituted for density in Equation 3.8 using

$$p = \rho r T, \qquad \text{Eq. 3.9}$$

where r is the gas constant and T is temperature. This gives the equation for pressure on the moving plate,

$$\nabla \bullet \left[\frac{h^3}{12\mu}p(\nabla p)\right] = pV + h\frac{\partial p}{\partial t}. \qquad \text{Eq. 3.10}$$

3.2 INCOMPRESSIBLE FLUID

In the case of an incompressible fluid, the pressure, p, on the surface of the moving plate is given by

$$\nabla \bullet \left(\frac{h^3}{12\mu}\nabla p\right) = V, \qquad \text{Eq. 3.11}$$

with the boundary condition $p=P_{ambient}$.

4. IMPLEMENTATION IN INTELLISUITE™

Commercial MEMS software packages have been developed to perform fully 3-dimensional dynamics analysis, allowing for analyses including RF switching time and natural frequency shift due to applied voltage bias [3]. In addition, recent advancements have enabled numerical simulation of micro-assembly techniques in MEMS devices [2] and post-assembly performance [4]. Each advance has allowed for the accurate computational analysis of a new family of MEMS devices. Now, with the ability to accurately and efficiently calculate the effect of squeezed film damping on the dynamic response, the accuracy of dynamic analyses of many devices has been improved.

If the assumptions listed above are valid, the velocity profile of the fluid between the moving plate and the solid surface can be approximated as parabolic in the thickness direction. In this case, the Navier-Stokes equations governing the fluid flow are reduced to a scalar equation in terms of the fluid pressure, as described. This squeezed film model is combined with a solid mechanical (finite-element) model in order to perform dynamic fluid-structure interaction analyses. In such an analysis, the solid mechanical model provides the plate geometry, location, and velocity to the squeezed film model. The squeezed film model, in turn, provides the resulting fluid pressure on the moving plate to the solid mechanical model.

In the implementation of squeezed film damping analysis in IntelliSuite™, the base plane must be fixed in space. That is, the film surface must have one moving plate and one fixed base plane; these are selected as a pair. Multiple pairs can be defined in the same analysis, and each pair has an independent set of fluid type, fluid properties, and ambient pressure specification.

The scalar equations (3.9 and 3.10) are discretized using quadratic finite elements, providing a fully 3-dimensional solution. For Equation 3.10, the Euler backward time integration scheme is used, and Newtonian iteration is applied at each step to achieve equilibrium.

In the presence of squeezed films, the solution of the dynamic structural response is limited to direct time integration with a fixed time increment. The time increment should be selected based on the characteristic periods of the structure. There is no stability restriction on the size of the time increment in the film pressure calculation.

For the parabolic velocity profile assumption to be applicable, the moving plate and the base plane should be relatively parallel, and the gap between them should be small compared to the width of the plate. The moving structure does not need to be rigid, however, becausethe gap height and vertical velocity are evaluated at each nodal point of the moving plate in the film pressure computation.

With the use of the stated assumptions, the computational expense required to solve the dynamic equations of motion in the presence of a fluid medium will be significantly reduced. As squeezed-film damping represents a major portion of microfluidic interaction for MEMS devices, this capability will allow MEMS design and development engineers to accurately simulate the performance of a large number of microfluidic MEMS devices.

5. EXAMPLES

Three examples will be presented to demonstrate the capabilities of squeezed film damping analysis. The first example is a solid plate moving in an incompressible fluid. The second example is a porous plate moving in an incompressible fluid. The third example is a solid plate moving in a compressible fluid.

5.1 SOLID PLATE IN INCOMPRESSIBLE FLUID

A square plate of width 400 μm and separated from the fixed base substrate at an initial height of 15 μm moves as a rigid body towards the base at a constant speed of 25000 μm/sec (25×10^{-3} m/sec), as shown in Figure 3.

The device is immersed in an incompressible liquid. The time history of the resulting pressure force on the moving plate for the period of 0 to 400 μsec is selected as the result for validation.

The parameters for this analysis are shown in Table 1.

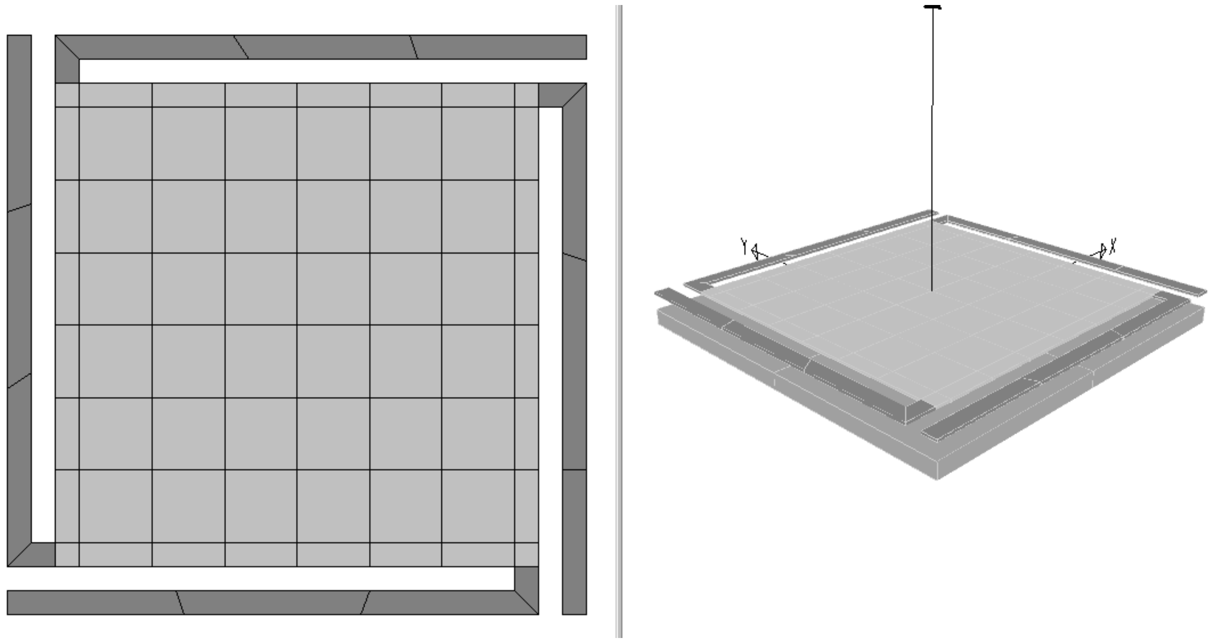

Figure 3: Solid square plate moving in incompressible liquid

Property	**Value**
Density of liquid, ρ	$1\text{x}10^3$ kg/m^3
Viscosity of liquid, μ	$1\text{x}10^{-3}$ N-sec/m^2
Ambient Pressure, $P_{ambient}$	0 Pa
Initial Gap Width	15 μm
Final Gap Width	5 μm

Table 1: Parameters for first example

The IntelliSuite squeezed film solution is compared to the transient Navier-Stokes solution. The comparison can be seen in Figure 4, showing the total fluid force on the plate as a function of time. The transient Navier-Stokes solution was calculated using the commercial CFD package Nekton [5].

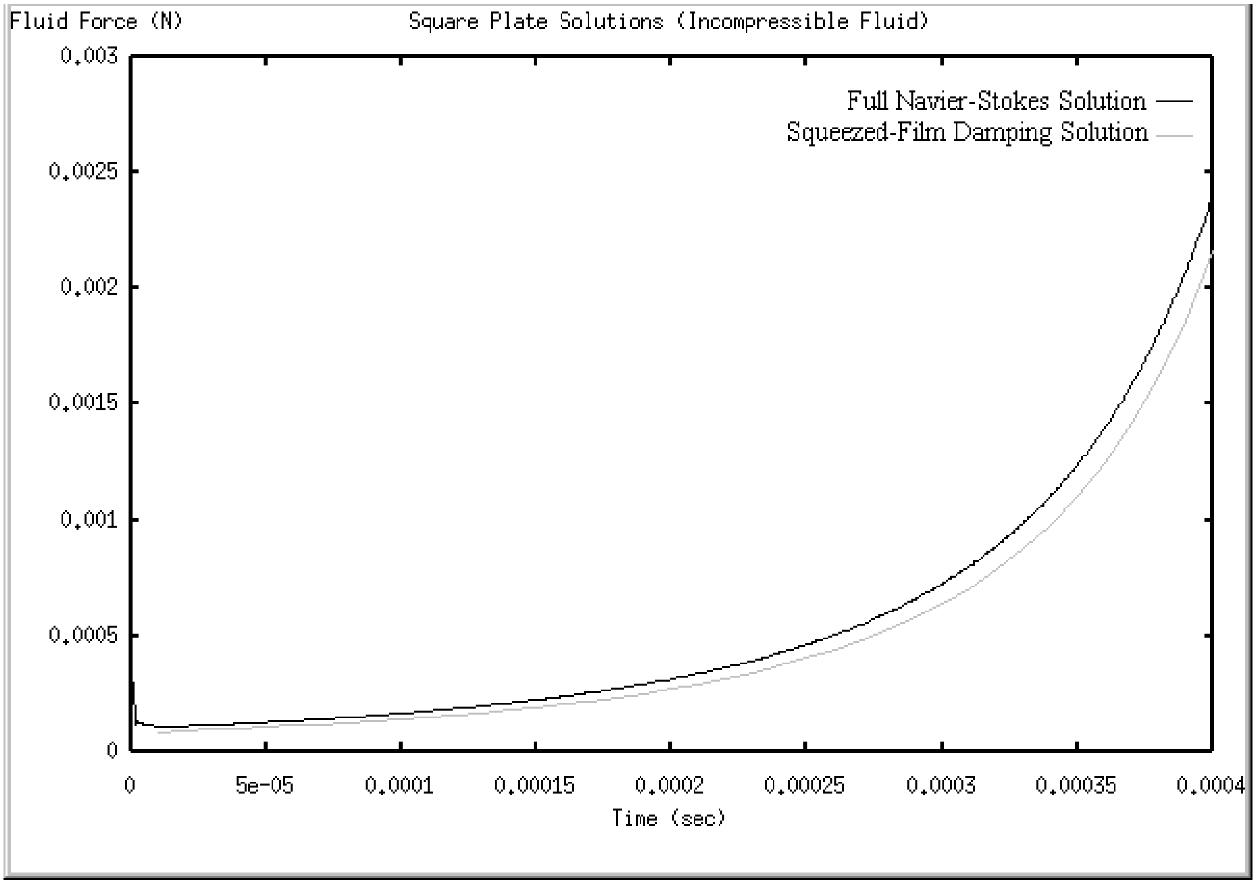

Figure 4: Comparison of fluid pressure force on a moving square plate calculated using squeezed-film damping and full Navier-Stokes CFD

By taking advantage of the stated assumptions, the solution time has been reduced by a factor of 10, and the calculated results are within 10% of the full Navier-Stokes solution.

5.2 POROUS PLATE IN INCOMPRESSIBLE FLUID

A square plate separated from the fixed base substrate at an initial height of 15 µm moves as a rigid body towards the base at a constant speed of 25000 µm/sec ($25x10^{-3}$ m/sec). The square plate has a width of 400 µm, but also has four square cutouts, each 60 µm in width, as shown in Figure 5.

The device is immersed in an incompressible liquid. The time history of the resulting pressure force on the moving plate for the period of 0 to 400 µsec is selected as the result for validation.

The parameters for this analysis are shown in Table 2.

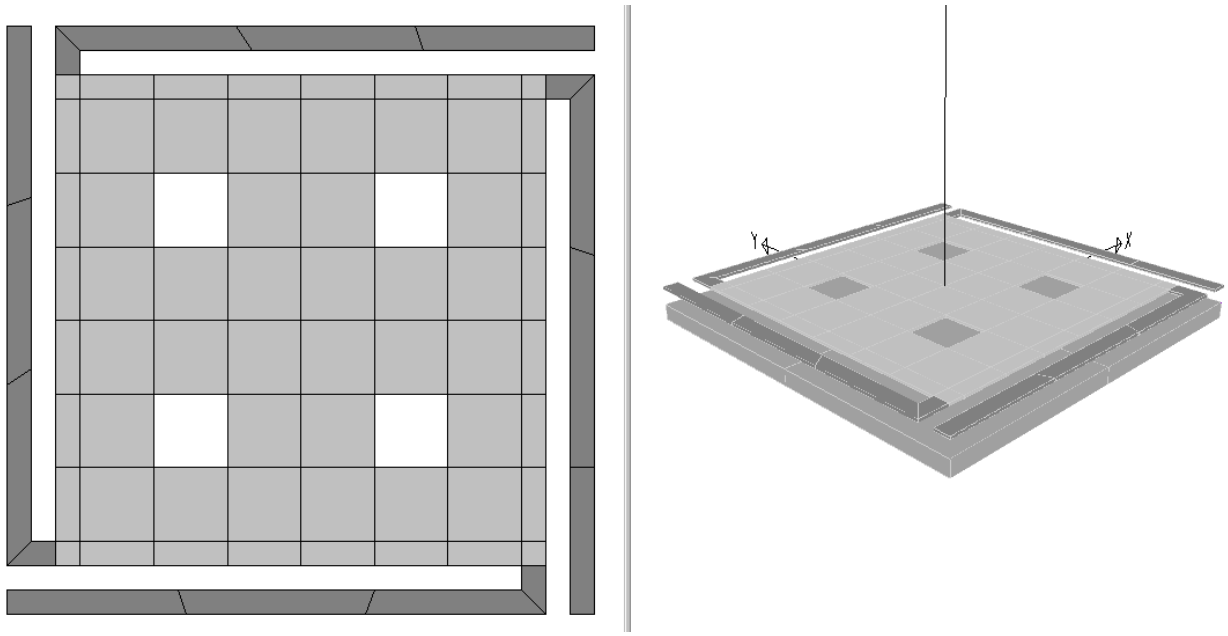

Figure 5: Square plate with cutouts moving in incompressible liquid

Property	**Value**
Density of liquid, ρ	1×10^{3} kg/m^{3}
Viscosity of liquid, μ	1×10^{-3} N-sec/m^{2}
Ambient Pressure, $P_{ambient}$	0 Pa
Initial Gap Width	15 μm
Final Gap Width	5 μm

Table 2: Parameters for second example

The IntelliSuite squeezed film solution is compared to the transient Navier-Stokes solution. The comparison can be seen in Figure 6, showing the total fluid force on the plate as a function of time. The transient Navier-Stokes solution was calculated using Nekton.

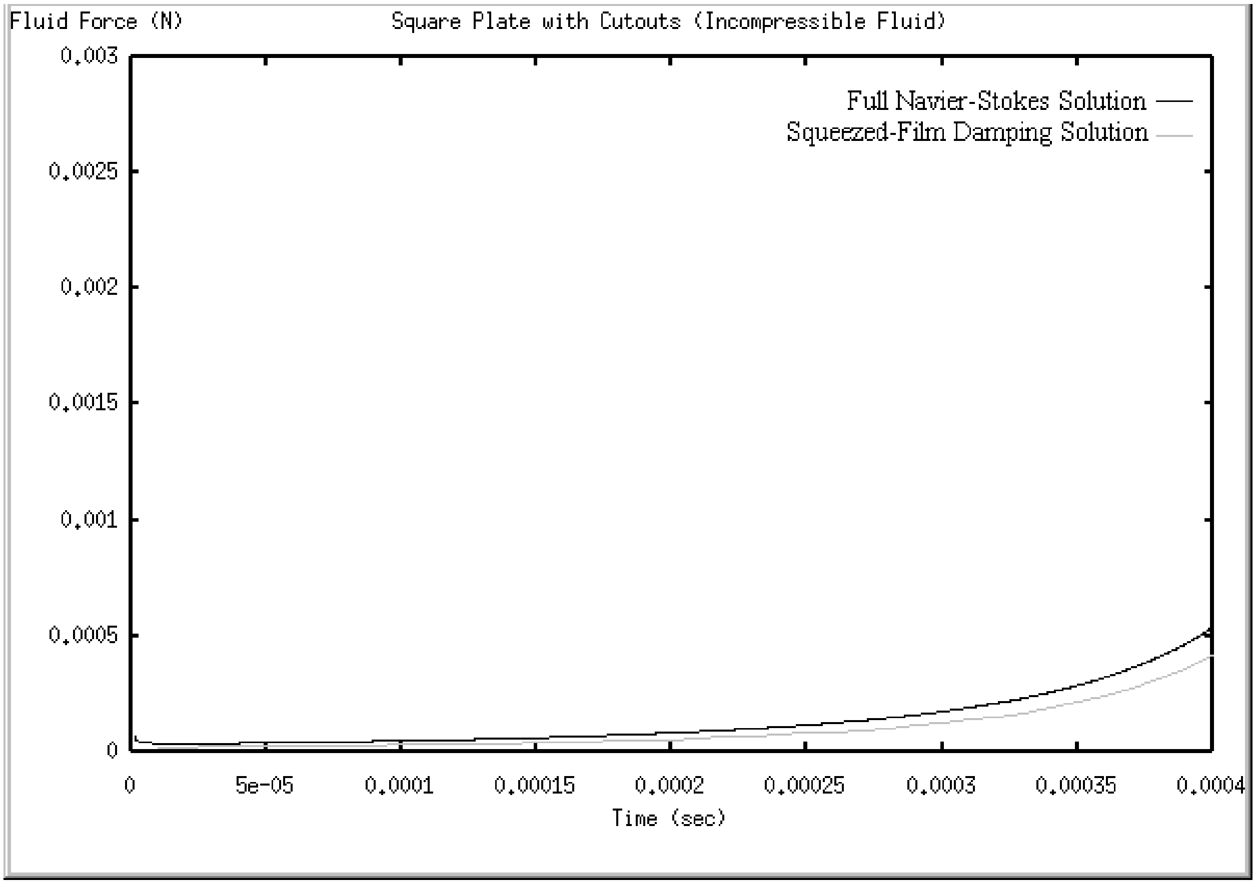

Figure 6: Comparison of fluid pressure force on a moving square plate with cutouts calculated using IntelliSuite and full Navier-Stokes CFD

Again, by taking advantage of the stated assumptions, the solution time has been reduced by a factor of 10, and the calculated results are within 10% of the full Navier-Stokes solution.

Comparison of Figure 4 and Figure 6 shows that the presence of cutouts, or pores, siginificantly alleviates the effects of squeezed film damping on the device.

5.3 SOLID PLATE IN COMPRESSIBLE FLUID

A solid square plate of width 400 µm is separated from the fixed base substrate at an average height of 10 µm. The plate undergoes small amplitude sinusoidal rigid-body oscillation normal to its plane, as shown in Figure 7.

The gap is filled with a compressible fluid (air at 1 atmosphere). The time history of the resulting pressure at the center of the moving plate for the period of 0 to 400 µsec is selected as the result for validation.

The parameters for this analysis are shown in Table 3.

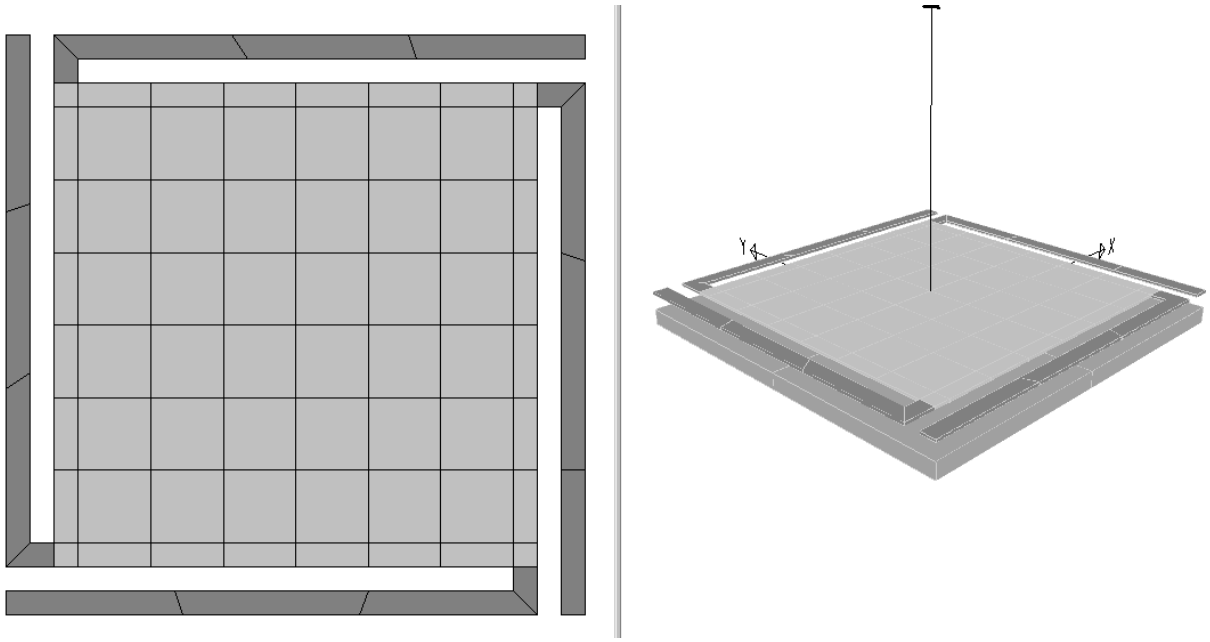

Figure 7: Square plate oscillates in ambient air

Property	**Value**
Density of fluid, ρ	1.2 kg/m^3
Viscosity of fluid, μ	1.8x10^{-5} N-sec/m^2
Ambient Pressure, $P_{ambient}$	0.1 MPa
Mean Gap Width	10 μm
Gap Amplitude	1 μm
Initial Gap Width	11 μm
Frequency of Oscillation	2500 Hz

Table 3: Parameters for third example

Using small amplitude approximation, the governing equation for the compressible squeezed film can be linearized with respect to the mean gap width and ambient pressure. The resulting equation is analogous to the transient heat conduction equation,

$$\nabla \bullet (k\nabla T) = q^B , \qquad \text{Eq. 5.1}$$

where k is thermal conductivity, T is temperature, and q^B is a heat source. The analogue between Equations 3.9 and 5.1 is clear. A reference solution is available for Equation 5.1.

The IntelliSuite squeezed film solution is compared to a transient heat transfer solution computed using Nekton. A comparison of the time history of the pressure at the center of the moving plate is given in Figure 8.

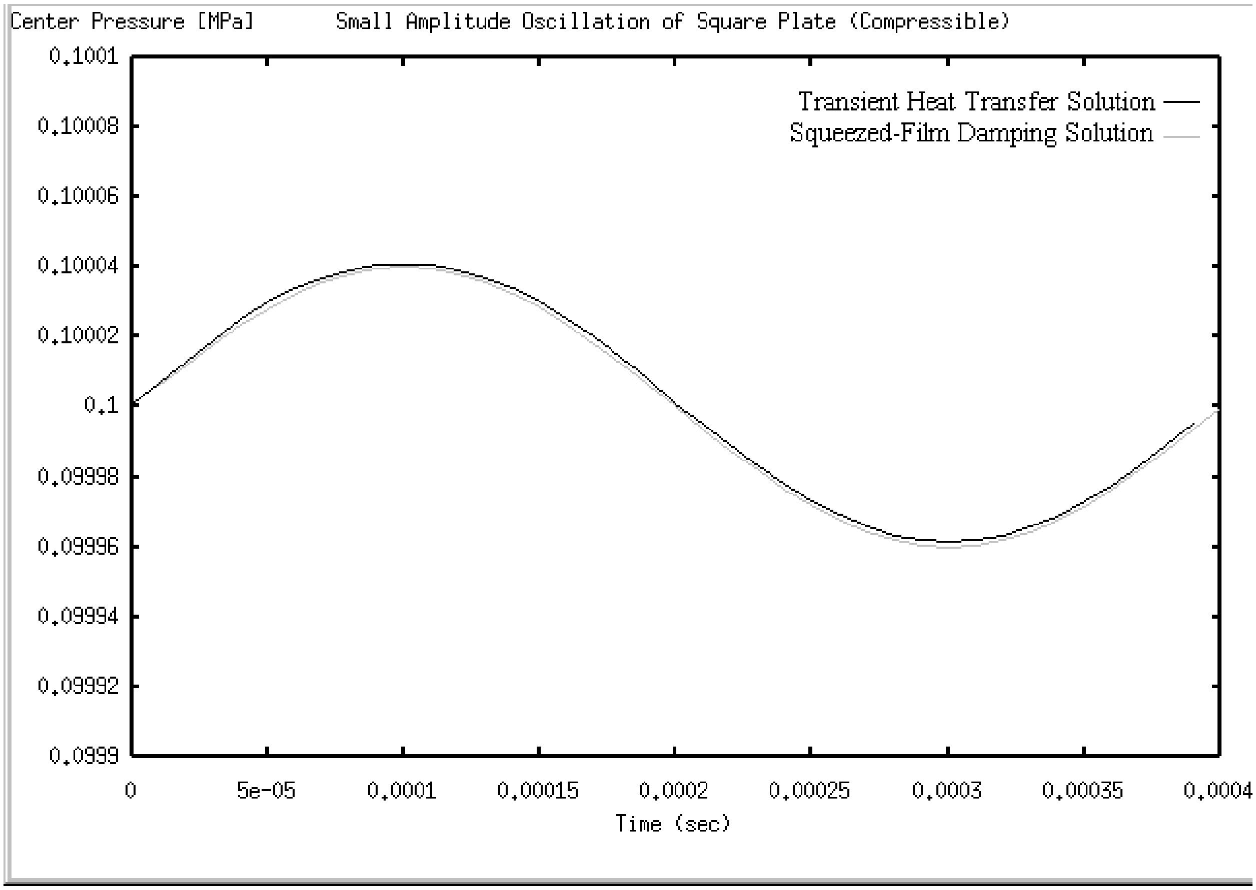

Figure 8: Comparison of the time history of pressure at the plate center.

In each of these three cases, the IntelliSuite squeezed film damping solution provided results within 10% accuracy of a full Navier-Stokes solution at a fraction (<10%) of the computational time.

6. CONCLUSIONS

A squeezed film damping analysis method was presented, including the governing equations and the required assumptions. If the flow can be assumed to be quasi-steady and viscous-dominant, and if the gap is much smaller than the plate width, the full Navier-Stokes equations can be simplified to provide an accurate solution in a reduced computation time. The governing equations were presented for both compressible and incompressible fluids.

Examples were presented comparing fully 3-dimensional squeezed-film solutions to full Navier-Stokes solutions for incompressible fluids and to an analogous transient heat transfer problem for the compressible fluid. In each case, the squeezed film algorithm provided results within 10% accuracy in less than 10% of the computational time.

7. REFERENCES

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