ACCURATE FULLY-COUPLED NATURAL FREQUENCY SHIFT OF MEMS ACTUATORS DUE TO VOLTAGE BIAS AND OTHER EXTERNAL FORCES

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ABSTRACT

This paper presents a new and general methodology for 3D simulation of natural frequency shift due to external forces. Specifically, the affect of applied voltage bias will be investigated. With this new method, which is a significant improvement over state-of-the-art 2D and reduced degree-of-freedom models, any class of electrostatically induced MEMS devices under dynamics investigation can be simulated with high accuracy.

This solution technique is the first reported 3D simulation for coupled frequency shift of MEMS devices. The computation of a 3D stiffness matrix associated with the electrostatic forces allows for the consideration of full 3D effects.

INTRODUCTION

A number of MEMS devices including gyros and accelerometers use external electrostatic forces as a means of tuning the device natural frequency or as a means of force feedback control [1]. By controlling the natural frequency of a device, one can actively control the sensitivity of the device to a broad band of excitation frequencies. Since there are coupled electrodynamic interactions, frequency tuning via electrostatic forces is a widely-used method and must be accurately predicted in design of the device.

Until now, CAD simulation of the effect of voltage bias on natural frequency has been limited to simplified lower degree-of-freedom macromodels or idealized 2-dimensional models [2] [3]. These models are applicable only to classes of devices where symmetric rigid-body motions, lumped parameter models, and reduced degree-of-freedom analysis can be assumed. Yet the effect of voltage on natural frequency shift is a 3D phenomena, and a general 3D model must be used when effects such as non-symmetric geometry, process induced warpage, and actuator levitation play an important role in the device behavior.

In the new simulation method presented here, the above modeling restrictions have been removed allowing for the accurate simulation of the natural frequency of a 3D model. Hence, the following limiting restrictions have been lifted:

- rigid-body assumption for movable region,

- lumped parameter modeling and reduced degrees-of-freedom motion, and

- a need to create databases or device-specific models in analysis

This paper will provide a technical description of the work along with CAD results from industrial MEMS devices.

TECHNICAL DESCRIPTION

CAD developments were made to fully take into account the influences of loads (thermal, electrostatic, and mechanical) as well as in-plane stress on the natural frequency of MEMS devices. These effects are simulated in a coupled 3D environment as part of IntelliSense's IntelliSuite™ system.

When taken into account, the above phenomena have a non-linear effect on the stiffness of the structure. By computing a modified stiffness matrix, the eigenvalue problem can be solved for the associated natural frequencies and mode shapes.

The approach used in this paper to model the natural frequency shift due to an applied voltage bias requires the computation of a stiffness matrix, Ke, associated with the electrostatic forces:

Single Degree-of-freedom System

Figure 1 shows the electrostatic force generated between two entities a voltage bias is applied. The electrostatic force changes non-linearly as the distance between the two entities decreases. This force displacement phenomenon can be modeled as an equivalent non-linear spring with negative stiffness.
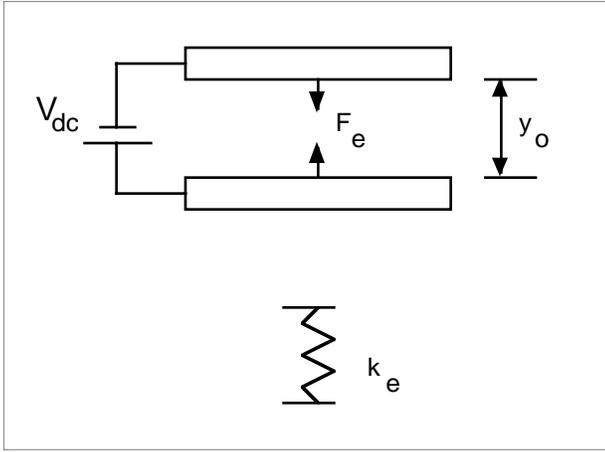
Figure 1: Electrostatic force modeled as an equivalent spring.

A linearized spring constant can be computed with the following equation

$$F_e = \frac{y_o}{2y^2} C_s V_{dc}^2 \qquad (1)$$

$$k_e = \left.\frac{\partial F_e}{\partial y}\right|_{y_o} = -\frac{C_s}{y_o^2} V_{dc}^2 \qquad (2)$$

For a one degree-of-freedom system, the natural frequency can be computed with the following equation:

$$\omega_y = \sqrt{\frac{k_m + k_e}{M}} \qquad (3)$$

Multiple Degree-of-freedom Systems

Extending the single degree-of-freedom approach to a general 3D multiple degree-of-freedom is challenging since the mechanical stiffness matrix must be modified and reconstructed to take into account the electrostatic results.

In order to solve the problem, an electrostatic boundary element solver is used to compute the electrostatic forces between entities. From these forces, an associated electrostatic spring matrix, Ke, is computed.

The distributed electrostatic force-displacement on a surface of a structure then can be related to the nonlinear spring (positive or negative) matrix, Ke, associated with the structure's surface. Algorithms have been developed to calculate the distributed electrostatic spring matrix, Ke.

It should be noted the structural displacement, D, is defined two dimensionally since it is a structure *surface* variable, however its 3D effect is fully taken into account with respect to the behavior of the model.

Ke is determined as follows: First, given two different perturbations around the balanced position, the 3D electrostatic forces on each panel of the structure surface for the two cases is calculated as follows:

Fep = fepx **i** + fepy **j** + fepz **k**; $\qquad (4)$

where

fep = sum (weight$_m$ * P$_{nm}$ * area$_{nm}$)

m = 1,... number of panels

n = 1. ... number of nodes

P is the electrostatic pressure.

The weight$_m$ variable indicates the weight which associates area m to point n. The spring matrix is calculated from the two different displacements and the 3D electrostatic force on each panel.

$$K_e = \frac{\partial F_e}{\partial D} \approx \frac{\Delta F_e}{\Delta D} \qquad (5)$$

In the case of electrostatically applied loads, the electrostatic stiffness matrix, K_e, can be combined with the stiffness matrix associated with the mechanical model, K_m, to obtain the total stiffness of the structure. Then the following eigenvalue problem can be solved for the natural frequencies, ω, and their associated mode shapes, U:

$$\omega^2 MU = (K_m + K_e)U \qquad (5)$$

RESULTS

Three examples are shown to demonstrate the power of the newly developed analysis of frequency shift due to voltage bias. The first example shown in Figure 2 is a cantilever beam. The creation of an accurate lumped model of this simple geometry would be very difficult since it has a deformable mass and is non-symmetric. A three dimensional analysis is necessary to take into account the torsional effects in the beam. The resulting graph shown in Figure 3 shows the resonant frequency shift for the first 2 modes as a function of voltage differential between the beam and ground plane.
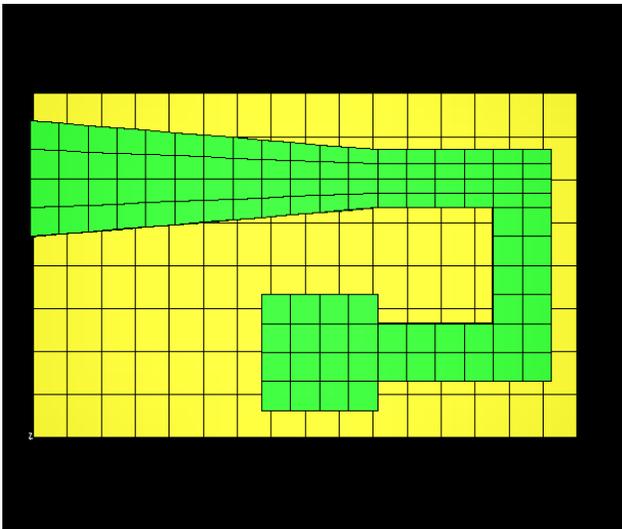

Figure 2: Top view of cantilever beam suspended 2 μm over ground plane. Left edge of beam is fixed.
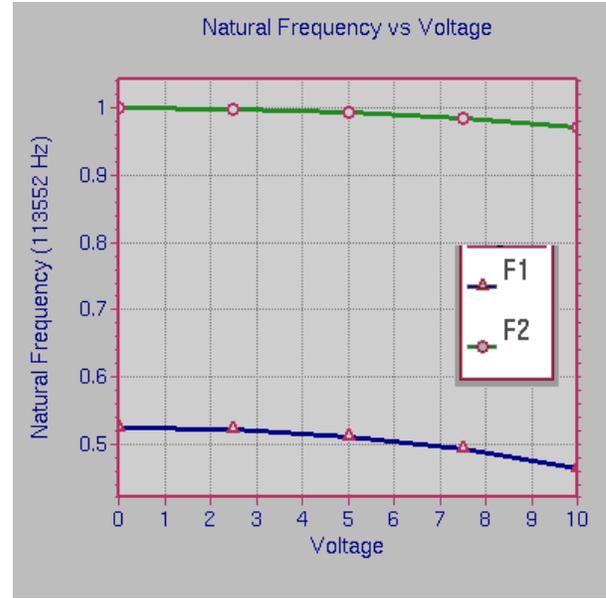

Figure 3: Frequency response due to voltage loads for structure shown in Figure 2.

The second example (Figure 4) is an actuator from industry with associated results (Figure 5).   The lower ground electrodes can be seen in the upper part of Figure 4. Voltage bias is applied between these lower electrodes and the upper deformable entity. Since this device has a non-rigid mass a 3D simulation must be used for accurate results.
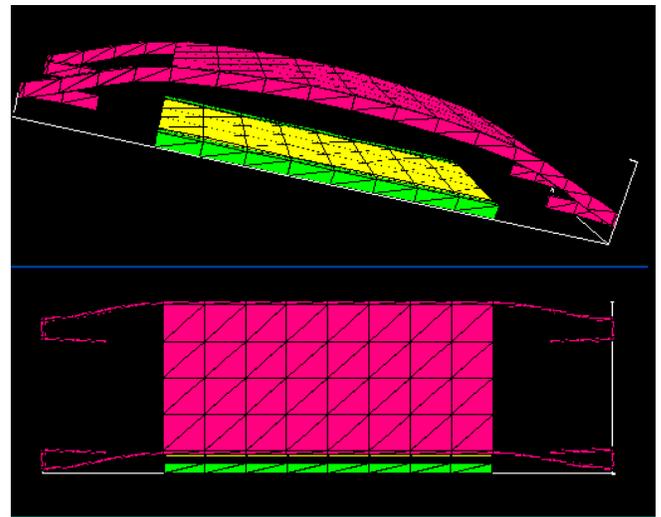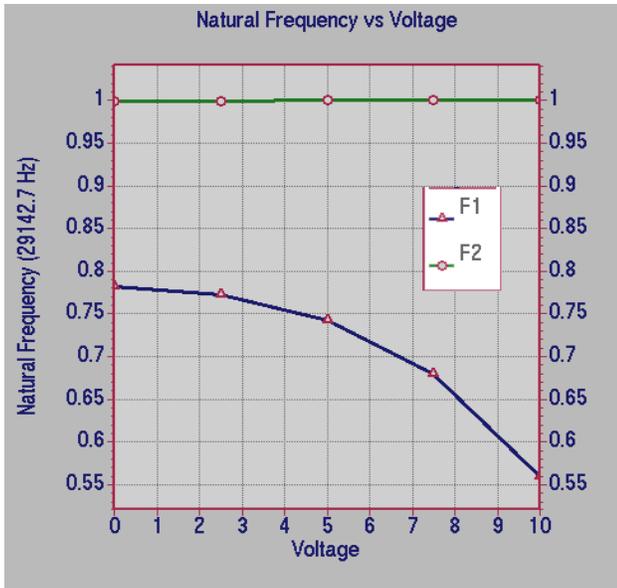

Figure 4: Shape of 1st mode of resonance (top), 2nd mode of resonance (bottom).

Figure 5: Frequency response due to voltage loads for structure shown in Figure 4.

Note that the first mode of resonance experiences a frequency decrease due to the fact that the motion of the first mode is in the direction of the electric field. The second mode does not experience a frequency shift since its motion is perpendicular to the electric field. In both examples, the levitation effect of the ground plane is taken into account.

The third example considered is an accelerometer shown in Figure 6. A proof mass is attached to eight support beams.
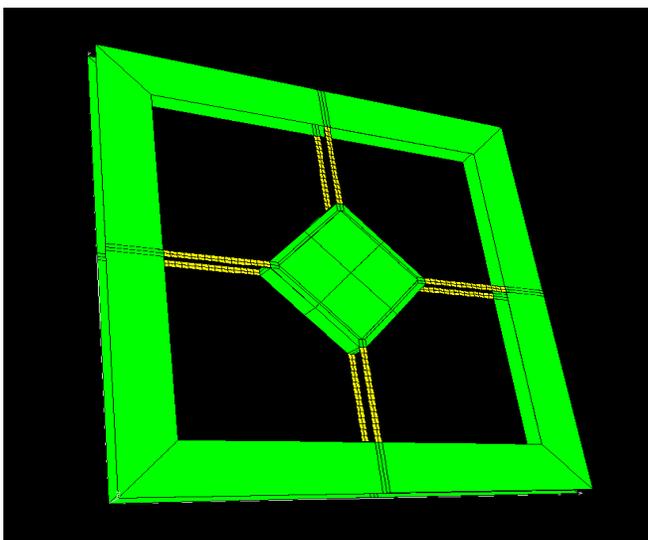
Figure 6: Accelerometer 3D model with finite element mesh (ground plane not shown).

An acceleration load is applied to the proof mass as a function of driving frequency. The acceleration load input is shown in Figure 7.
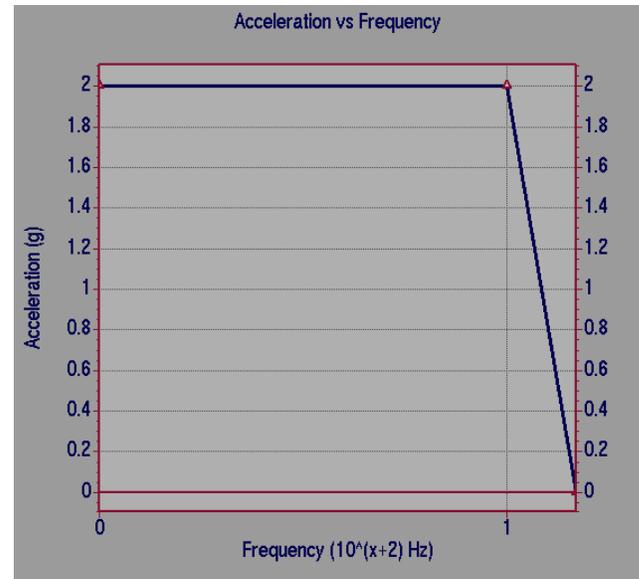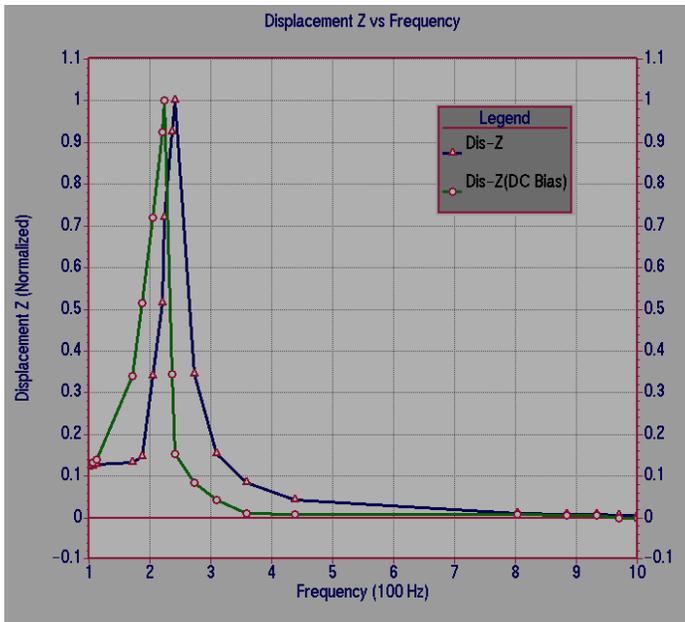

Figure 7: Acceleration as a function of frequency applied to the proof mass.

Two situations were considered : (I) voltage bias and (II) no voltage bias applied between the proof mass and the ground plane.

A harmonic steady-state analysis was performed which shows the response of the proof mass displacement as the function of frequency, the results of which are shown in Figure 8. In this frequency range, only the first mode of resonance is excited.

In this dynamic analysis the critical damping factor associated with the first mode was taken as 0.05. The natural frequency (with and without DC bias) and associated mode shape are first calculated. Then a mode-based harmonic steady-state analysis is performed to obtain the results in Figure 8.

Note the frequency shift of the proof mass over the range of frequencies simulated.

Figure 8: Normalized displacement as a function of excitation frequency. Acceleration and DC bias loads taken into account in 3D.

CONCLUSIONS

Based on the above method for determining the natural frequency shift due to voltage bias, a broad general range of electrostatically actuated MEMS devices can now be simulated accurately. This progress reported is an important improvement over the current state-of-the art. Three dimensional natural frequency analysis of MEMS devices associated dynamics is now possible.

REFERENCES

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