LAGRANGE LOOK TABLE BASED APPROACH FOR EFFICIENT DYNAMIC MODELING OF ELECTROSTATIC INERTIAL DEVICES

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Introduction

Accurate modeling of electrostatic gyroscopes requires properly captured far field electrostatic effects including parasitic capacitances, relevant high order modes (which can severely effect performance), and fluidic damping effects such as Couette damping and Squeeze film damping. The accurate coupled modeling of the gyro performance over mechanical, electrostatic, and fluidic domains comes at a computational cost — a typical FEA/BEA based transient simulation may take a few hours to a few days depending upon the model complexity.

Designers also need high fidelity models to couple with electrical circuit simulators like SPICE, or other VHDL-AMS/Verilog-A based simulators to model the total system performance. Typically, reduced order models or macro-models representing the device are used at this stage, but each method has its own limitations. Models based upon simplified mass, spring, and damper representations are insufficient because they do not accurately capture all of the higher order mode effects, parasitic capacitances, fluidic damping effects, electrostatic levitation effects, etc. Simplified hierarchical models such as NODAS, SUGAR or SYNPLE [1-3] are useful for quick exploration of the design space, but often can't fully capture second order effects. Arnoldi and Krylov based model order reduction techniques and energy based techniques based have been previously reported as methods used to create dynamic models of MEMS devices [4-5]. However, such techniques are typically useful for only linear problems and difficult to implement in a general fashion.

We present a new method to automatically capture the total energy in the system at multiple operating points. The total strain and electrostatic energy of each important mode is captured. A novel lookup table based approach is used to create 3D system models. The system models have been implemented in various SPICE and HDL formats. For the first time, this allows analysts to perform accurate 3D dynamic simulations coupled with a circuit simulator.

Implementation

The method for the system model extraction within IntelliSuite is based upon Lagrangian Mechanics. Lagrangian mechanics provides a structured approach which accounts for the total energy of all physical domains within a holonomic system and automatically derives the equations of motion with full coupling effects [6-7]. The general procedure for calculating the Lagrangian function is as follows:

1. Choose the generalized co-ordinates of the system q_j. In the case of a discretized (meshed) MEMS device, the eigenshapes (Φ_j), eigenfrequency (ω_j), and generalized mass (m_j) of the device are easy to compute. The mode shapes are chosen as the basis functions or the generalized co-ordinates of the system. The analyst is further given the ability to intelligently identify the relevant/dominant modes of the system. By our estimate, 95% of the energy is typically contained within 2-3 dominant modes of the system.

2. Compute the kinetic energy (T) and potential energy (V) of each of the energy fields within the system. The Lagrange function is defined as: $L = T\text{-}V$. In the case of electrostatically actuated MEMS, the electrical potential energy is readily derived from the capacitance between various entities in the system. The electrostatic potential energy is computed from the system capacitance matrix $[C_{ks}]$ derived using a boundary element formulation as previously reported [8]. The mutual capacitance energy as a function of modal amplitudes is calculated and stored in a lookup table. In addition, the damping coefficient (ξ_j) of each mode is also calculated using the technique previously reported by IntelliSense [9].

3. Determine the generalized forces in the system, $\Xi q_j = \Sigma\, F_j\,(\partial r_j/\partial q_j)$, based upon the displacements (∂r_j) in the system arising from the non-conservative work (∂W) performed in the system. In the case of a MEMS or a purely mechanical device, this can be derived from the principal of virtual work. The strain energy function (∂W_{st}) of the system can be determined for each mode as it scales through to a predefined maximum displacement in the system. The strain energy function is evaluated for each of the modes, and the strain energy for each of the modes as it spans the design space (strain energy as a function of modal amplitude) is stored in a second lookup table.

The equations of motion can be derived as:

$$\frac{d}{dt}\frac{\partial L}{\partial q'_j} - \frac{\partial L}{\partial q_j} = \Xi_{qj}$$

$$m_j \ddot{q}_j + 2\xi_j \omega_j m_j \dot{q}_j + \frac{\partial W_{st}}{\partial q_j} = \frac{1}{2}\sum_r \frac{\partial C_{ks}}{\partial q_j}\cdot(V_k - V_s)^2 + \sum_{i=1}^{n}\Phi'_j\cdot F_i$$

Once these analyses are complete, the information is stored in a final look up table that can be used in a system level simulator. Extrapolation and interpolation within the lookup table can be used to determine the energy associated with a particular excitation of the device. Within the system level simulator, different loading conditions can be applied to the system model to analyze how it will react, allowing optimization of the device and its control structure.

The algorithm for extracting the basis functions and the Lagrangian described above have been implemented in IntelliSuite's TEM analysis module. The equations of motion based upon the Lagrangian are automatically translated into a SPICE/HDL model and are implemented in IntelliSense's SYNPLE software.

Results

Micro-g accelerometer

Figure 1 (a) shows a finite element 3D model (IntelliSuite) of an SOI based capacitive accelerometer intended for μg sensing for seismic applications. It is used to measure the acceleration and the acceleration response is measured in terms of change of capacitance between the electrodes. The two rectangular blue structures in the middle are the electrodes. The capacitance change between the inertial mass and the electrodes is recorded to sense the acceleration. The macromodel was extracted using IntelliSuite's System Model Extraction (SME) module and the system model is simulated in SYNPLE.

Figure 1(b) shows the dynamic response of the accelerometer to a 1 ms, 1g acceleration pulse. The difference between the FEA and the Lagrangian SME model is less than 2%. The FEA simulation took approximately 20 minutes to complete as opposed to 10 seconds for the SME model.
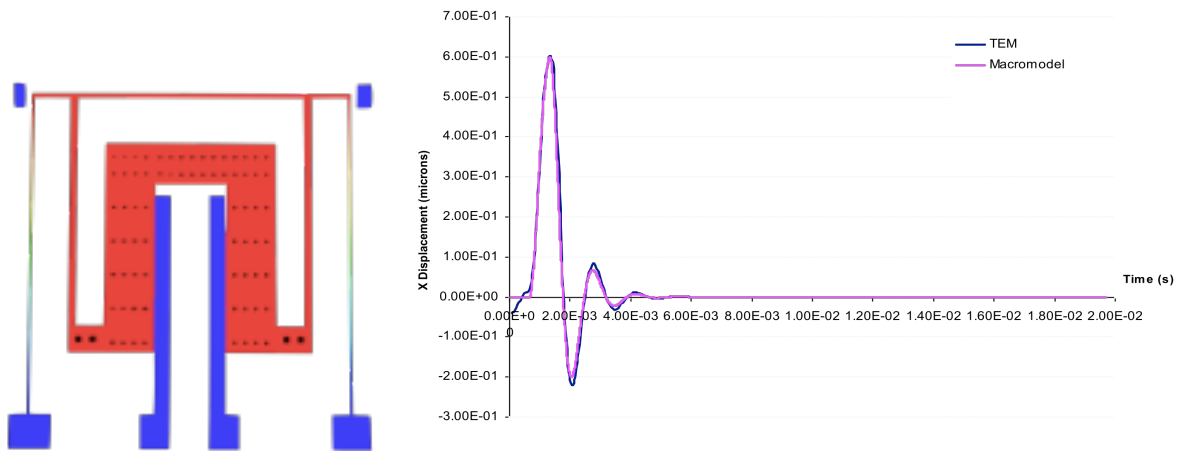
Figure 1 (a) 3D FEA model (top view) of an μg-accelerometer (b) comparison of the dynamic response of the accelerometer, to a 1 ms 1g loading, between FEA and SME calculations.

Inertial gyroscope

Figure 2 below shows an inertial grade gyroscope ($< 0.1°$/hr rtHz) gyro being developed at Georgia Tech [10].
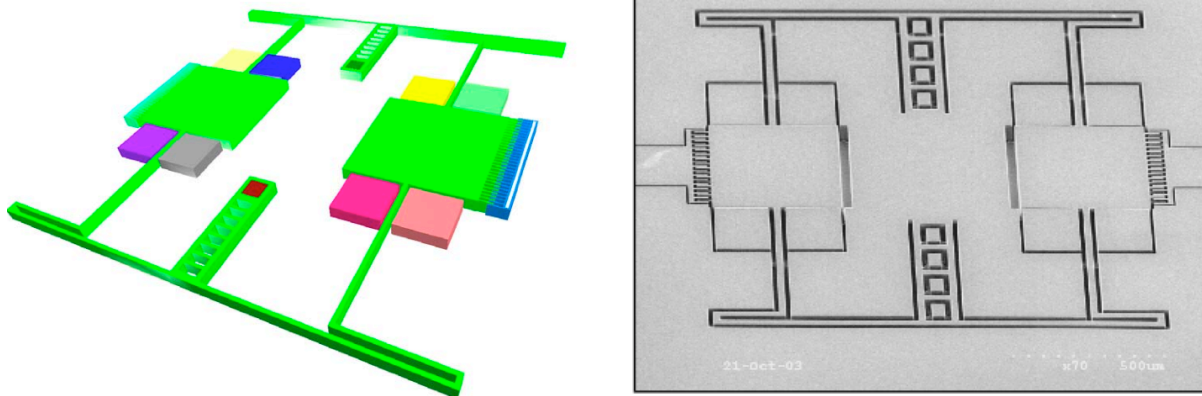
Figure 2(a-b): Inertial grade gyro FEA model and fabricated device. Picture courtesy Zaman et al Georgia Tech University.

The startup response of the gyro is shown in the Figure 3(a-b) below. The x-displacement (drive motion) and the y-displacement (sense motion due to Coriolis force) are plotted as a function of time. As can be seen, the displacements are exactly out of phase. The response of the gyro to a sinusoidal rotational input is shown in the Figure 3(c)
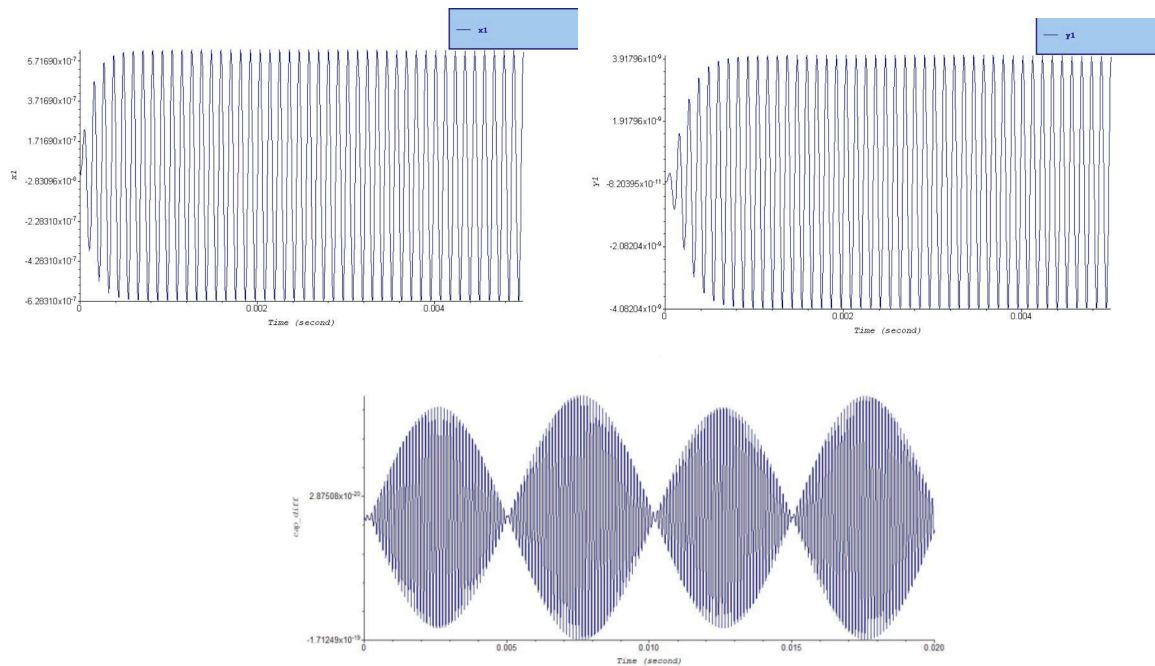

Figure 3(a-c): (a-b). Startup response of the gyro (c) gyro response to a sinusoidal rotational input

The FEA/BEA multiphysics model took nearly 24 hours to compute the first 5 cycles of the gyro response. Computing the first 50 cycles of the startup response would take nearly 10 days. The Lagrangian model took about 4 hours to extract. Once extracted, the Lagrangian results matched

the coupled FEA/BEA to within 1% and took approximately 30 seconds to compute on a modern desktop PC. The response to the sinusoidal input took less than 1 minute to compute.

One of the major advantages of this approach is the seamless and efficient integration of the device model with circuit simulation. The analyst is able to co-simulate the device and the associated ASIC. Figure 4 below shows the device model combined with a transistor level Sigma-Delta read-out circuit and the digital output of the gyroscope to a rotational sinusoidal input.
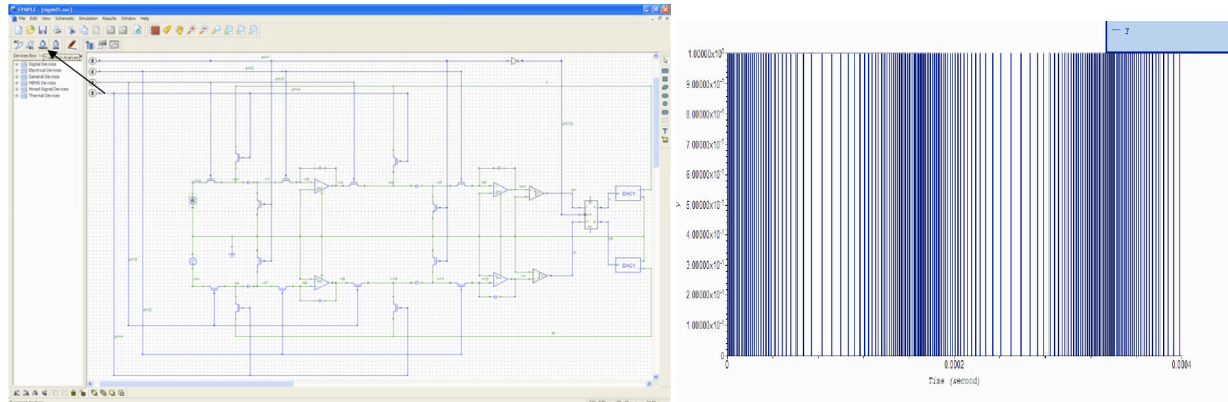
Figure 4: (a) Transistor level modeling of the gyro and its associated read-out circuit (b) digital output stream of the gyro response to a sinusoidal rotational input.

Conclusions

We have demonstrated a mode preserving Lagrangian based approach for efficient modeling of electrostatic MEMS devices such as accelerometers and gyroscopes. Without loss of generality, the approach can be used for other MEMS devices such as microphones, pressure sensors, micro-mirrors and other sensors and actuators. The simplicity of this approach lends itself to modeling other multi-physical domains, like those found in magnetic, piezoelectric, or thermal devices. The technique has been implemented in a commercial tool.

Since the Lagrangian is derived from the FEA model in form of a lookup table, the accuracy of the model matches that of the FEA. The time-savings associated with this approach are apparent. Simulations which take hours to days can now be performed in a matter of seconds to minutes. In addition, the Lagrangian based approach is amenable to co-simulation of the MEMS and electronics.

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